$$Z = 2.42 \begin{cases} > Z_{.05} = 1.96 \text{ (two tailed test)} \\ < Z_{.01} = 2.58 \text{ (two tailed test)} \end{cases}$$

The **null hypothesis** that there is no difference between the mean life time of bulbs **is rejected at 5% level but not at 1% level of significance.**

The null hypothesis is rejected at both the levels of significance in a one tailed test as the respective critical values are 1.645 and 2.33.

---

26. *The mean of two large samples lof 1000 and 2000 members are 168.75 cms and 170 cms respectively. Can the samples be regarded as drawn from the same population of standard deviation 6.25 cms. ?*

\>> $\bar{x}_1 = 168.75$, $\bar{x}_2 = 170$

$n_1 = 1000$, $n_2 = 2000$

$$Z = \frac{\bar{x}_2 - \bar{x}_1}{\sigma \sqrt{1/n_1 + 1/n_2}}$$

$$= \frac{1.25}{6.25\sqrt{1/1000 + 1/2000}} = 5.16$$

$Z = 5.16$ is very much greater than $Z_{.05} = 1.96$ and also $Z_{.01} = 2.58$. Thus we say that the difference between the sample means is significant and **we conclude that the samples cannot be regarded as drawn from the same population.**

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27. *A random sample for 1000 workers in company has mean wage of Rs 50 per day and S.D of Rs 15. Another sample of 1500 workers from another company has mean wage of Rs 45 per day and S.D of Rs 20. Does the mean rate of wages varies between the two companies ? Find the 95 % confidence limits for the difference of the mean wages of the population of the two companies.*

\>> Company - 1 : $\bar{x}_1 = 50$, $\sigma_1 = 15$, $n_1 = 1000$

Company - 2 : $\bar{x}_2 = 45$, $\sigma_2 = 20$, $n_2 = 1500$

$$Z = \frac{(\bar{x}_1 - \bar{x}_2)}{\sqrt{\sigma_1^2/n_1 + \sigma_2^2/n_2}}$$

ie $$Z = \frac{5}{\sqrt{225/1000 + 400/1500}} = \frac{5}{0.7012} = 7.1306$$

$Z = 7.1306$ is greater than $Z_{.05} = 1.96$ and $Z_{.01} = 2.58$.

Hence we can say that the **difference between the mean wages is significant both at 5 % and 1 % levels of significance.**

Also 95 % confidence limits for the difference of mean wages is given by

$$(\bar{x}_1 - \bar{x}_2) \pm 1.96 \sqrt{\sigma_1^2 / n_1 + \sigma_2^2 / n_2}$$

$$= 5 \pm 1.96\,(0.7012)$$

$$= 5 \pm 1.374$$

$= 3.626$ and 6.374 or 3.63 and 6.37 approximately.

Thus we can say with 95 % confidence that the difference of population mean of wages between the two companies lies between Rs 3.63 and Rs 6.37

Test of significance for difference of properties.

28. *In an exit poll enquiry it was revealed that 600 voters in one locality and 400 voters from an other locality favoured 55% and 48% respectively a particular party to come to power. Test the hypothesis that there is a difference in the locality in respect of the opinion.*

>> By data, $p_1 = \dfrac{55}{100} = 0.55$ (First locality)

$p_2 = \dfrac{48}{100} = 0.48$ (Second locality)

H_0 is the null hypothesis that there is no difference in the locality.

Population proportion $p = \dfrac{n_1 p_1 + n_2 p_2}{n_1 + n_2}$ where $n_1 = 600,\ n_2 = 400$

$$\therefore \quad p = \frac{600\,(0.55) + 400\,(0.48)}{600 + 400} = 0.522$$

Also $q = 1 - p = 0.478$

Consider $Z = \dfrac{p_1 - p_2}{\sqrt{pq\,(1/n_1 + 1/n_2)}}$

i.e., $$Z = \frac{0.55 - 0.48}{\sqrt{(0.522)(0.478)(1/600 + 1/400)}} = 2.171$$

$$Z = 2.171 \begin{cases} > Z_{.05} = 1.96 \text{ (Two tailed test)} \\ < Z_{.01} = 2.58 \text{ (Two tailed test)} \end{cases}$$

Thus the null hypothesis that there is no difference between the localities is rejected at 5% level but not at 1% level of significance.

---

29. *One type of aircraft is found to develop engine trouble in 5 flights out of a total of 100 and another type in 7 flights out of a total of 200 flights. Is there a significant difference in the two types of aircrafts so far as engine defects are concerned ?*

>> Let p_1 and p_2 be the proportion of defects in the two types of aircrafts.

$$\therefore \quad p_1 = \frac{5}{100} = 0.05, \ p_2 = \frac{7}{200} = 0.035$$

H_0 is the null hypothesis that there is no significant difference between the two type of aircrafts.

Combined proportion = $p = \frac{5+7}{100+200} = \frac{12}{300} = 0.04$ and $q = 1 - p = 0.96$

$$\text{Consider } Z = \frac{p_1 - p_2}{\sqrt{pq\,(1/n_1 + 1/n_2)}}$$

$$= \frac{0.05 - 0.035}{\sqrt{(0.04)(0.96)\left(1/100 + 1/200\right)}} = 0.625$$

$$Z = 0.625 \begin{cases} < Z_{.05} = 1.96 \text{ (two tailed test)} \\ < Z_{.01} = 2.58 \text{ (two tailed test)} \end{cases}$$

Thus the null hypothesis is accepted both at 5 % and 1 % levels of significance.

---

30. *Random sample of 1000 engineering students from a city A and 800 from city B were taken. It was found that 400 students in each of the sample were from payment quota. Does the data reveal a significant difference between the two cities in respect of payment quota students?*

>> $n_1 = 1000,\ n_2 = 800$

$$p_1 = \frac{400}{1000} = 0.4 \ ; \ p_2 = \frac{400}{800} = 0.5$$

$$\therefore \quad p = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2} = \frac{1000(0.4) + 800(0.5)}{1800} = \frac{4}{9}$$

$$\therefore \quad q = 1 - p = 5/9$$

Let H_0 be the null hypothesis that there is no significant difference between the 2 cities.

$$\text{Consider } Z = \frac{p_2 - p_1}{\sqrt{p\,q\,(1/n_1 + 1/n_2)}}$$

$$Z = \frac{0.1}{\sqrt{4/9 \times 5/9\,(1/1000 + 1/800)}} = 4.243$$

$$Z = 4.243 > \begin{cases} Z_{.05} = 1.96 \\ Z_{.01} = 2.58 \end{cases}$$

Thus the hypothesis H_0 is rejected both at 5% and 1% levels of significance.

Student's t distribution / test

31. *Find the student's t for the following variable values in a sample of eight : −4, −2, −2, 0, 2, 2, 3, 3, taking the mean of the universe to be zero.*

\>> $t = \dfrac{\bar{x} - \mu}{s}\sqrt{n}$

By data $\mu = 0$ and we have $n = 8$

$$\bar{x} = \frac{1}{8}(-4-2-2+0+2+2+3+3) = \frac{1}{4} = 0.25$$

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

$$= \frac{1}{7}\Big\{(-4.25)^2 + (-2.25)^2 + (-2.25)^2 + (-0.25)^2 + (1.75)^2 + (1.75)^2 + (2.75)^2 + (2.75)^2\Big\}$$

$$s^2 = \frac{1}{7}(49.5) = 7.07 \qquad \therefore \quad s = 2.66$$

Thus $t = \dfrac{0.25 - 0}{2.66}\sqrt{8} = \mathbf{0.266}$

Note : *The expression for s^2 can also be put in the following form.*

$$s^2 = \frac{1}{n-1}\left\{\sum_1^n (x_i^2 - 2x_i\bar{x} + \bar{x}^2)\right\}$$

$$= \frac{1}{n-1}\left\{\sum_1^n x_i^2 - 2\sum_1^n x_i \cdot \frac{1}{n}\sum_1^n x_i + n\left(\frac{1}{n}\sum x_i\right)^2\right\}$$

$$= \frac{1}{n-1}\left\{\sum_1^n x_i^2 - \frac{2}{n}(\sum x_i)^2 + \frac{1}{n}(\sum x_i)^2\right\}$$

$$\therefore \quad s^2 = \frac{1}{n-1}\left\{\sum_1^n x_i^2 - \frac{1}{n}(\sum x_i)^2\right\}$$

According to this formula we have in the given example

$$s^2 = \frac{1}{7}\left\{50 - \frac{1}{8}(2)^2\right\} = \frac{1}{7}(49.5) = 7.07$$

Remark : *We can employ this formula when $\bar{x}$ is not an integer.*

---

32. *A machine is expected to produce nails of length 3 inches. A random sample of 25 nails gave an average length of 3.1 inch with standard deviation 0.3. Can it be said that the machine is producing nails as per specification? ($t_{0.05}$ for 24 d.f is 2.064)*

\>> By data we have,

$$\mu = 3,\ \bar{x} = 3.1,\ n = 25,\ s = 0.3$$

$$t = \frac{\bar{x}-\mu}{s}\sqrt{n} = \frac{0.1}{0.3}\sqrt{25} = 1.67 < 2.064$$

Thus the **hypothesis** that the machine is producing nails as per specification is **accepted at 5% level of significance.**

---

33. *Ten individuals are chosen at random from a population and their heights in inches are found to be 63, 63, 66, 67, 68, 69, 70, 70, 71, 71. Test the hypothesis that the mean height of the universe is 66 inches. ($t_{.05} = 2.262$ for 9 d.f)*

\>> We have $\mu = 66,\ n = 10$

$$\bar{x} = \frac{\sum x}{n} = \frac{678}{10} = 67.8$$

$$s^2 = \frac{1}{n-1}\sum(x-\bar{x})^2$$

$$s^2 = \frac{1}{9}\left[(63-67.8)^2 + \cdots + (71-67.8)^2\right] = 9.067 \quad \therefore \quad s = 3.011$$

We have $t = \frac{\bar{x}-\mu}{s}\sqrt{n} = \frac{(67.8-66)}{3.011}\sqrt{10} = 1.89 < 2.262$

Thus the hypothesis is accepted at 5% level of significance.

---

34. *A sample of 10 measurements of the diameter of a sphere gave a mean of 12cm and a standard deviation 0.15cm. Find the 95% confidence limits for the actual diameter.*

\>> By data $n = 10,\ \bar{x} = 12,\ s = 0.15$

Also $t_{.05}$ for $9\ d \cdot f = 2.262$

Confidence limits for the actual diameter is given by

$$\bar{x} \pm \left[\frac{s}{\sqrt{n}}\right] t_{.05} = 12 \pm \frac{0.15}{\sqrt{10}}(2.262) = 12 \pm 0.1073$$

Thus 11.893cm to 12.107cm is the confidence limits for the actual diameter.

---

35. *A certain stimulus administered to each of the 12 patients resulted in the following change in blood pressure.* 5, 2, 8, −1, 3, 0, 6, −2, 1, 5, 0, 4. *Can it be concluded that the stimulus will increase the blood pressure?* ($t_{.05}$ *for* 11 $d \cdot f$ = 2.201)

>> $$\bar{x} = \frac{\sum x}{n} = \frac{31}{12} = 2.5833$$

$$s^2 = \frac{1}{n-1}\sum(x-\bar{x})^2 = \frac{1}{n-1}\left\{\sum x^2 - \frac{1}{n}(\sum x)^2\right\}$$

$$s^2 = \frac{1}{11}\left\{185 - \frac{1}{12}(31)^2\right\} = 9.538 \quad \therefore \; s = 3.088$$

We have, $$t = \frac{\bar{x}-\mu}{s}\sqrt{n}$$

Let us suppose that the stimulus administration is not accompanied with increase in blood pressure, we can take $\mu = 0$

$$\therefore \quad t = \frac{2.5833-0}{3.088}\sqrt{12} = 2.8979 \approx 2.9 > 2.201$$

Hence the hypothesis is rejected at 5% level of significance. We conclude with 95% confidence that the stimulus in general is accompanied with increase in blood pressure.

---

36. *A group of boys and girls were given an intelligence test. The mean score, S.D score and numbers in each group are as follows.*

	Boys	*Girls*
Mean	74	70
SD	8	10
n	12	10

Is the difference between the means of the two groups significant at 5% level of significance ($t_{.05}$ = 2.086 *for* 20 $d \cdot f$)

>> We have by data $\bar{x} = 74,\ s_1 = 8,\ n_1 = 12$ [Boys]

$\bar{y} = 70,\ s_2 = 10,\ n_2 = 10$ [Girls]

Also we have $$t = \frac{\bar{x}-\bar{y}}{s\sqrt{1/n_1 + 1/n_2}}$$

where $s^2 = \dfrac{1}{n_1 + n_2 - 2}\left\{ \sum_{i=1}^{n_1} (x_i - \bar{x})^2 + \sum_{j=1}^{n_2} (y_j - \bar{y})^2 \right\}$

or $s^2 = \dfrac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}$

Now $s^2 = \dfrac{12(64) + 10(100)}{20} = \dfrac{1768}{20} = 88.4 \quad \therefore \quad s = 9.402 \approx 9.4$

Hence $t = \dfrac{74 - 70}{9.4\sqrt{1/12 + 1/10}} = 0.994$

$t = 0.994 < t_{.05} = 2.086$

Thus the hypothesis that there is a difference between the means of the two groups is accepted at 5% level of significance.

---

37. *A sample of 11 rats from a central population had an average blood viscosity of 3.92 with a standard deviation of 0.61. On the basis of this sample, establish 95% fiducial limits for μ the mean blood viscosity of the central population ($t_{.05} = 2.228$ for 10 d.f)*

>> By data $\bar{x} = 3.92,\ s = 0.61,\ n = 11$

95% fiducial limits for μ are $\bar{x} \pm \dfrac{s}{\sqrt{n}}\, t_{.05}$

i.e., $= 3.92 \pm \dfrac{0.61}{\sqrt{11}}(2.228)$

$= 3.92 \pm 0.41 = 3.51$ and 4.33

Thus 95% confidence limits for μ are 3.51 and 4.33.

---

38. *Two types of batteries are tested for their length of life and the following results were obtained.*

Battery A : $n_1 = 10,\ \bar{x}_1 = 500$ *hrs* , $\sigma_1^2 = 100$

Battery B : $n_2 = 10,\ \bar{x}_2 = 500$ *hrs* , $\sigma_2^2 = 121$

Compute Student's t and test whether there is a significant difference in the two means.

>> $s^2 = \dfrac{n_1\sigma_1^2 + n_2\sigma_2^2}{n_1 + n_2 - 2}$

$s^2 = \dfrac{(10 \times 100) + (10 \times 121)}{18} = 122.78 \quad \therefore \quad s = 11.0805$

We have, $$t = \frac{(\bar{x}_2 - \bar{x}_1)}{s\sqrt{1/n_1 + 1/n_2}}$$

$$t = \frac{60}{11.0805\sqrt{0.1 + 0.1}} = 12.1081 \approx 12.11$$

This value of t is greater than the table value of t for 18 $d \cdot f$ at all levels of significance.

The null hypothesis that there is no significant difference in the two means is rejected at all significance levels.

---

39. *A group of 10 boys fed on a diet A and another group of 8 boys fed on a different diet B for a period of 6 months recorded the following increase in weights (lbs.)*

Diet A :	5	6	8	1	12	4	3	9	6	10
Diet B :	2	3	6	8	10	1	2	8		

Test whether diets A and B differ significantly regarding their effect on increase in weight.

>> Let the variable x correspond to the diet A and y to the diet B.

$$\bar{x} = \frac{\Sigma x}{n_1} = \frac{64}{10} = 6.4 \; ; \; \bar{y} = \frac{\Sigma y}{n_2} = \frac{40}{8} = 5$$

$$\sum_1^{n_1} (x - \bar{x})^2 = 102.4 \; ; \; \sum_1^{n_2} (y - \bar{y})^2 = 82$$

$$s^2 = \frac{1}{n_1 + n_2 - 2} \left\{ \sum_1^{n_1} (x - \bar{x})^2 + \sum_1^{n_2} (y - \bar{y})^2 \right\}$$

$$s^2 = \frac{1}{16}(102.4 + 82) = \frac{184.4}{16} = 11.525 \quad \therefore \quad s = 3.395$$

Consider $t = \dfrac{\bar{x} - \bar{y}}{s\sqrt{1/n_1 + 1/n_2}}$

$$t = \frac{1.4}{3.395\sqrt{1/10 + 1/8}} = 0.86935 \approx 0.87$$

But $t_{.05}$ for $16\, d \cdot f = 2.12$ from the tables. $t = 0.87$ is less than the table value for $16\, d \cdot f$ at 5% level of significance.

Thus we conclude that the two diets do not differ significantly regarding their effect on increase in weight.

---

40. *Two horses A and B were tested according to the time (in seconds) to run a particular race with the following results.*

Horse A : 28 30 32 33 33 29 34

Horse B : 29 30 30 24 27 29

Test whether you can discriminate between the two horses.

>> Let the variables x and y respectively correspond to horse A and horse B.

$$\bar{x} = \frac{\sum x}{n_1} = \frac{219}{7} = 31.3\,,\quad \bar{y} = \frac{\sum y}{n_2} = \frac{169}{6} = 28.2$$

$$\sum_{1}^{n_1} (x-\bar{x})^2 = 31.43\,,\quad \sum_{1}^{n_2} (y-\bar{y})^2 = 26.84$$

$$s^2 = \frac{1}{n_1+n_2-2}\left\{\sum_{1}^{n_1} (x-\bar{x})^2 + \sum_{1}^{n_2} (y-\bar{y})^2\right\}$$

$$s^2 = \frac{1}{11}\,(31.43+26.84) = 5.2973 \quad \therefore \quad s = 2.3016$$

Consider $t = \dfrac{\bar{x}-\bar{y}}{s\,\sqrt{1/n_1+1/n_2}}$

$$t = \frac{(31.3-28.2)}{2.3016\,\sqrt{1/7+1/6}} = 2.42$$

But $\quad t_{.05} = 2.2$ and $t_{.02} = 2.72$ for $11\, d\cdot f$

$$t = 2.42 \begin{cases} > t_{.05} = 2.2 \\ < t_{.02} = 2.72 \end{cases}$$

The discrimination between the horses is significant at 5% level but not at 2% level of significance.

Chi-Square distribution

41. *A die is thrown 264 times and the number appearing on the face (x) follows the following frequency distribution.*

x	1	2	3	4	5	6
f	40	32	28	58	54	60

Calculate the value of χ^2.

>> The frequencies in the given data are the observed frequencies. Assuming that the dice is unbiassed the expected number of frequencies for the numbers 1, 2, 3, 4, 5, 6 to appear on the face is $\frac{264}{6} = 44$ each. Now the data is as follows :

No. on the dice	1	2	3	4	5	6
Observed frequency (0_i)	40	32	28	58	54	60
Expected frequency (E_i)	44	44	44	44	44	44

$$\chi^2 = \Sigma \frac{(0_i - E_i)^2}{E_i}$$

$$= \frac{(40-44)^2}{44} + \frac{(32-44)^2}{44} + \cdots + \frac{(60-44)^2}{44}$$

$$= \frac{1}{44}\left[16 + 144 + 256 + 196 + 100 + 256\right] = \frac{968}{44} = 22$$

Thus $\chi^2 = \mathbf{22}$

---

42. *Five dice were thrown 96 times and the numbers 1, 2, or 3 appearing on the face of the dice follows the frequency distribution as below.*

No. of dice showing 1,2 or 3	5	4	3	2	1	0
Frequency	7	19	35	24	8	3

Test the hypothesis that the data follows a binomial distribution. ($\chi^2_{0.05} = 11.07$ *for 5 d.f*)

>> The data gives the observed frequencies and we need to calculate the expected frequencies.

Probability of a single dice throwing 1, 2 or 3 is $p = 3/6 = 1/2 \quad \therefore \quad q = 1 - p = 1/2$

The binomial distribution of fit is, $N(q+p)^n = 96\left(1/2 + 1/2\right)^5$

The theoretical frequencies of getting 5, 4, 3, 2, 1, 0 successes with 5 dice are respectively the successive terms of the binomial expansion.

They are respectively $96 \times \frac{1}{2^5}, 96 \times 5C_1 \times \frac{1}{2^5}, \ldots 96 \times \frac{1}{2^5}$ or 3, 15, 30, 30, 15, 3.

We have the table of observed and expected frequencies.

0_i	7	19	35	24	8	3
E_i	3	15	30	30	15	3

$$\chi^2 = \Sigma \frac{(0_i - E_i)^2}{E_i}$$

$$= \frac{16}{3} + \frac{16}{15} + \frac{25}{30} + \frac{36}{30} + \frac{49}{15} + \frac{0}{3} = 11.7$$

$$\chi^2 = 11.7 > \chi^2_{0.05} = 11.07$$

Thus the hypothesis that the data follows a binomial distribution is rejected.

---

43. *A sample analysis of examination results of 500 students was made. It was found that 220 students had failed, 170 had secured third class 90 had secured second class and 20 had secured first class. Do these figures support the general examination result which is in the ratio 4 : 3 : 2:1 for the respective categories ($\chi^2_{0.05} = 7.81$ for 3 d.f)*

>> Let us take the hypothesis that these figures support to the general result in the ratio 4 : 3 : 2 : 1.

The expected frequencies in the respective category are

$$\frac{4}{10} \times 500, \frac{3}{10} \times 500, \frac{2}{10} \times 500, \frac{1}{10} \times 500 \quad \text{or} \quad 200, 150, 100, 50.$$

We have the following table.

0_i	220	170	90	20
E_i	200	150	100	50

$$\chi^2 = \Sigma \frac{(0_i - E_i)^2}{E_i}$$

$$= \frac{400}{200} + \frac{400}{150} + \frac{100}{100} + \frac{900}{50}$$

$$\chi^2 = 23.67 > \chi^2_{0.05} = 7.81$$

Thus the hypothesis is rejected.

---

44. *4 coins are tossed 100 times and the following results were obtained. Fit a binomial distribution for the data and test the goodness of fit ($\chi^2_{0.05}$ = 9.49 for 4 d.f)*

Number of heads	0	1	2	3	4
Frequency	5	29	36	25	5

>> Referring to Problem-28 in Unit - VII, we have obtained the theoretical frequencies equal to 7, 26, 37, 24, 6 respectively.

We have the following table.

0_i	5	29	36	25	5
E_i	7	26	37	24	6

$$\chi^2 = \Sigma \frac{(0_i - E_i)^2}{E_i}$$

$$= \frac{4}{7} + \frac{9}{26} + \frac{1}{37} + \frac{1}{24} + \frac{1}{6} = 1.15$$

$$\chi^2 = 1.15 < \chi^2_{0.05} = 9.49$$

Thus the hypothesis that the fitness is good can be accepted.

---

45. *Fit a Poisson distribution for the following data and test the goodness of fit given that* $\chi^2_{0.05} = 7.815$ *for 3 d.f*

x	0	1	2	3	4
f	122	60	15	2	1

>> Referring to Problem-32 in Unit - VII we have obtained the theoretical frequencies equal to 121, 61, 15, 3, 0. Since the last of the expected frequency is 0 we shall club it with the previous one.

We have the following table.

0_i	122	60	15	$2+1 = 3$
E_i	121	61	15	$3+0 = 3$

$$\chi^2 = \Sigma \frac{(0_i - E_i)^2}{E_i}$$

$$\chi^2 = \frac{1}{121} + \frac{1}{61} + 0 + 0 = 0.025$$

$\chi^2 = 0.025 < \chi^2_{0.05} = 7.815$. The fitness is considered good.

Thus the hypothesis that the fitness is good can be accepted.

---

46. *The number of accidents per day (x) as recorded in a textile industry over a period of 400 days is given below. Test the goodness of fit in respect of Poisson distribution of fit to the given data* ($\chi^2_{0.05} = 9.49$ *for 4 d.f*)

x	0	1	2	3	4	5
f	173	168	37	18	3	1

>> Referring to the Problem-33 in Unit- VII, the corresponding theoretical frequencies are 183, 143, 56, 15, 3, 0. We shall club the last two frequencies to have the following table.

0_i	173	168	37	18	$3+1 = 4$
E_i	183	143	56	15	$3+0 = 3$

$$\chi^2 = \Sigma \frac{(0_i - E_i)^2}{E_i}$$

$$\chi^2 = \frac{100}{183} + \frac{625}{143} + \frac{361}{56} + \frac{9}{15} + \frac{1}{3} = 12.297 \approx 12.3$$

$\chi^2 = 12.3 > \chi^2_{0.05} = 9.49$. The fitness is not good.

Thus the hypothesis that the fitness is good is rejected.

EXERCISES

1. A random sample of size 2 is drawn from the population 3, 4, 5. Find the sampling distribution of the sample mean. (a) with replacement (b) without replacement. Find the sample mean and sample variance in these two cases.

2. 500 ball bearings have a mean weight of 142.30 gms. and S.D of 8.5 gms. Find the probability that a random sample of 100 ball bearings chosen from this group will have a combined weight (a) between 14,061 and 14,175 gms. (b) more than 14,460 gms.

3. The weights of packages received by a department store have a mean of 136 kgs. and a S.D of 22.5 kgs. What is the probability that 25 packages received at random and loaded on an elevator will exceed the safety limit of the elevator quoted as 3720 kgs.

4. A 'die' was thrown 9000 times and a throw of 5 or 6 was obtained 3240 times. On the assumption of random throwing, do the data indicate an unbiassed die ?

5. A sample of 900 days is taken from meteorological records of a certain district and 100 of them are found to be foggy. Find the 99.73% confidence level probable limits of the percentage of foggy days in the district.

6. The mean and S.D marks of a sample of 100 students are 67.45 and 2.92 respectively. Find a) 95% b)99% confidence intervals for estimating the mean marks of the population.

7. The mean of samples of size 1000 and 2000 are 67.5 cms. and 68 cms. respectively. Can the samples be regarded as drawn from the same population of S.D 2.5 cms ?

8. A machine produced 20 defective units in a sample of 400. After over oiling the machine it produced 10 defective in a batch of 300. Has the machine improved due to over oiling ?

9. Ten individuals are chosen at random from a population and their heights in inches are found to be 63, 63, 64, 65, 66, 69, 69, 70, 70, 71. Discuss the suggestion that the mean height of the population is 65 inches given that $t_{.05} = 2.262$ for $9\,d \cdot f$

10. From a random sample of 10 pigs fed on diet A, the increase in weight in a certain period were 10, 6, 16, 17, 13, 12, 8, 14, 15, 9 lbs. For another random sample of 12 pigs fed on diet B, the increase in weight in the same period were 7, 13, 22, 15, 12, 14, 18, 8, 21, 23, 10, 7 lbs. Test whether diets A and B differ significantly regarding their effect on increase in weight. ($t_{.05}$ for $20\,d \cdot f$ is equal to 2.09)

11. 4 coins were tossed 160 times and the following results were obtained.

No. of heads	0	1	2	3	4
frequency	17	52	54	31	6

Test the goodness of fit of the binomial distribution.
($\chi^2_{0.05} = 9.49$ for 4 d.f)

12. Fit a Poisson distribution for the following data and test the goodness of fit given that $\chi^2_{0.05} = 9.49$ for 4 d.f

x	0	1	2	3	4
f	419	352	154	56	19

ANSWERS

1. (a) 4 , 1/3 (b) 4 , 1/6
2. (a) 0.2222 (b) 0.0013
3. 0.0023
4. $Z = 5.4$ and the hypothesis is rejected at 1% level of significance.
5. 7.96% to 14.26%
6. (a) 66.88 and 68.02 (b) 66.7 and 68.2
7. $Z = 5.1$; Samples cannot be regarded as drawn from the same population.
8. $Z = 0.4254$. Hypothesis is accepted at 5% level of significance.
9. $t = 2.02$; Hypothesis is accepted at 5% level of significance.
10. $t = 1.6$, difference is not significant. The two diets do not differ significantly regarding increase in weight.
11. Fitness is not good.
12. Fitness is good.

BEATING THE MEMORY

[Formulae, Properties and Results to be remembered from all the units at a glance]

PART - A

Unit-I Numerical Methods - 1

Formulae for solving the initial value problem :

$$\frac{dy}{dx} = f(x, y);\ y(x_0) = y_0. \text{ To compute } y(x_0 + h).$$

➢ *Picard's formula*

$$y = y_0 + \int_{x_0}^{x} f(x, y)\, dx$$

First approximation : $y_1 = y_0 + \int_{x_0}^{x} f(x, y_0)\, dx$

Second approximation : $y_2 = y_0 + \int_{x_0}^{x} f(x, y_1)\, dx$ etc.,

➢ *Taylor's series*

$$y(x) = y(x_0) + (x - x_0)\, y'(x_0) + \frac{(x - x_0)^2}{2!}\, y''(x_0) + \cdots$$

➢ *Modified Euler's formula* [M.E.F]

Taking $x_1 = x_0 + h$ and $y_1 = f(x_1)$

$y_1^{(0)} = y_0 + hf(x_0, y_0)$ ... [Initial approx. / Euler's formula]

$y_1^{(1)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(0)}) \right]$... [First approx./ M.E.F]

$y_1^{(2)} = y_0 + \frac{h}{2}\, [f(x_0, y_0) + f(x_1, y_1^{(1)})]$... [Second approx. / M.E.F]

➢ *Runge - Kutta formula*

$$y(x_0+h) = y_0 + \frac{1}{6}[k_1+2k_2+2k_3+k_4] \text{ where}$$

$$k_1 = hf(x_0, y_0)$$

$$k_2 = hf\left(x_0+\frac{h}{2}, y_0+\frac{k_1}{2}\right)$$

$$k_3 = hf\left(x_0+\frac{h}{2}, y_0+\frac{k_2}{2}\right)$$

$$k_4 = hf(x_0+h, y_0+k_3)$$

➢ *Predictor and Corrector formulae*

Data : $\frac{dy}{dx} = f(x, y)$; $y(x_0) = y_0$, $y(x_1) = y_1$, $y(x_2) = y_2$, $y(x_3) = y_3$

where $x_1 = x_0+h$, $x_2 = x_0+2h$, $x_3 = x_0+3h$

Further $x_4 = x_0+4h$ and $y_4 = y(x_4)$ is to be computed.

➢ *Milne's predictor and corrector formulae*

$$y_4^{(P)} = y_0 + \frac{4h}{3}(2y_1' - y_2' + 2y_3') \ldots \text{[Predictor formula]}$$

$$y_4^{(C)} = y_2 + \frac{h}{3}(y_2' + 4y_3' + y_4') \ldots\ldots \text{[Corrector formula]}$$

➢ *Adams - Bashforth predictor and corrector formula*

$$y_4^{(P)} = y_3 + \frac{h}{24}(55y_3' - 59y_2' + 37y_1' - 9y_0') \ldots \text{[Predictor formula]}$$

$$y_4^{(C)} = y_3 + \frac{h}{24}(9y_4' + 19y_3' - 5y_2' + y_1') \ldots\ldots \text{[Corrector formula]}$$

Unit - II Numerical Methods - 2

Numerical solution of simultaneous first order ODEs

Data : $\frac{dy}{dx} = f(x, y, z)$, $\frac{dz}{dx} = g(x, y, z)$

Initial condition $y(x_0) = y_0$, $z(x_0) = z_0$ $[y = y_0, z = z_0, x = x_0]$

➢ *Picard's formula*

$$y = y_0 + \int_{x_0}^{x} f(x, y, z)\, dx \;;\; z = z_0 + \int_{x_0}^{x} g(x, y, z)\, dx$$

First approx. $y_1 = y_0 + \int_{x_0}^{x} f(x, y_0, z_0)\, dx \;;\; z_1 = z_0 + \int_{x_0}^{x} g(x, y_0, z_0)\, dx$

Second approx. $y_2 = y_0 + \int_{x_0}^{x} f(x, y_1, z_1)\, dx \;;\; z_2 = z_0 + \int_{x_0}^{x} g(x, y_1, z_1)\, dx$ etc.,

➢ *Runge - Kutta formula*

$$k_1 = hf(x_0, y_0, z_0) \quad ; \quad l_1 = hg(x_0, y_0, z_0)$$

$$k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}, z_0 + \frac{l_1}{2}\right) \quad ; \quad l_2 = hg\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}, z_0 + \frac{l_1}{2}\right)$$

$$k_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}, z_0 + \frac{l_2}{2}\right) \quad ; \quad l_3 = hg\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}, z_0 + \frac{l_2}{2}\right)$$

$$k_4 = hf(x_0 + h, y_0 + k_3, z_0 + l_3) \quad ; \quad l_4 = hg(x_0 + h, y_0 + k_3, z_0 + l_3)$$

The required $y(x_0 + h) = y_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

and $z(x_0 + h) = z_0 + \frac{1}{6}(l_1 + 2l_2 + 2l_3 + l_4)$

Numerical solution of 2nd order ODEs by Picard's method & Runge-Kutta method.

Data : $y'' = g(x, y, y')$; $y(x_0) = y_0$, $y'(x_0) = y_0'$

Put $y' = z$ which gives $y'' = z'$.

This will give a system of equations (Simultaneous equations)

$\frac{dy}{dx} = z$ and $\frac{dz}{dx} = g(x, y, z)$ with the initial conditions $y(x_0) = y_0$ and $z(x_0) = z_0$ where y_0' is denoted by z_0.

Taking $f(x, y, z) = z$, the system of equations are in the form.

$$\frac{dy}{dx} = f(x, y, z),\ \frac{dz}{dx} = g(x, y, z) \;;\; y = y_0,\ z = z_0,\ x = x_0$$

The solution is obtained as earlier.

➢ *Milne's method*

➲ We put $y' = z$ which gives $y'' = \frac{dz}{dx} = z'$.
The given d.e becomes $z' = f(x, y, z)$

➲ We equip with the following table of values.

x	x_0	x_1	x_2	x_3
y	y_0	y_1	y_2	y_3
$y' = z$	$y_0' = z_0$	$y_1' = z_1$	$y_2' = z_2$	$y_3' = z_3$
$y'' = z'$	$y_0'' = z_0'$	$y_1'' = z_2'$	$y_2'' = z_2'$	$y_3'' = z_3'$

➲ We first apply predictor formula to compute $y_4^{(P)}$ and $z_4^{(P)}$ where,

$$y_4^{(P)} = y_0 + \frac{4h}{3}(2z_1 - z_2 + 2z_3), \text{ since } y' = z.$$

$$z_4^{(P)} = z_0 + \frac{4h}{3}(2z_1' - z_2' + 2z_3')$$

➲ We compute $z_4' = f(x_4, y_4, z_4)$ and then apply corrector formula where,

$$y_4^{(C)} = y_2 + \frac{h}{3}(z_2 + 4z_3 + z_4)$$

$$z_4^{(C)} = z_2 + \frac{h}{3}(z_2' + 4z_3' + z_4')$$

➲ Corrector formula can be applied repeatedly for better accuracy.

Unit - III Complex Variables - 1

Complex number

➢ *Cartesian form* : $z = x + iy$ where $i = \sqrt{-1}$ or $i^2 = -1$

➢ *Polar form* : $z = re^{i\theta}$ where we have $e^{i\theta} = \cos\theta + i\sin\theta$

➢ *Modulus of* z : $|z| = r = \sqrt{x^2 + y^2}$

➢ *Amplitude / Argument of* z : *amp* z / *arg* $z = \theta = \tan^{-1}(y/x)$

➢ *Complex conjugate* : $\bar{z} = x - iy$

➢ *Some important results :*

(i) $\cos\theta = \dfrac{e^{i\theta}+e^{-i\theta}}{2}$; $\sin\theta = \dfrac{e^{i\theta}-e^{-i\theta}}{2i}$

(ii) $\cos(i\theta) = \cosh\theta$; $\sin(i\theta) = i\sinh\theta$

➢ *Complex valued function*

$w = f(z) = f(x+iy) = u(x, y)+iv(x, y)$ [Cartesian form]

$w = f(z) = f(re^{i\theta}) = u(r, \theta)+iv(r, \theta)$ [Polar form]

➢ *Analytic (Regular, Holomorphic) function*

$f'(z) = \dfrac{dw}{dz} = \underset{\delta z \to 0}{lt} \dfrac{f(z+\delta z)-f(z)}{\delta z}$ exists and is unique.

➢ *Cauchy - Riemann (C – R) equations*

(i) Cartesian form : $u_x = v_y$ and $v_x = -u_y$

(ii) Polar form : $r u_r = v_\theta$ and $r v_r = -u_\theta$

➢ *Derivative of the analytic function*

(i) Cartesian form : $f'(z) = u_x + i v_x$

(ii) Polar form : $f'(z) = e^{-i\theta}(u_r + i v_r)$

➢ *Harmonic function*

$\nabla^2\phi = 0$ implies that ϕ is harmonic.

(i) Cartesian form : $\dfrac{\partial^2\phi}{\partial x^2} + \dfrac{\partial^2\phi}{\partial y^2} = 0$

(ii) Polar form : $\dfrac{\partial^2\phi}{\partial r^2} + \dfrac{1}{r}\dfrac{\partial\phi}{\partial r} + \dfrac{1}{r^2}\dfrac{\partial^2\phi}{\partial\theta^2} = 0$

➢ *Applications to flow problems*

- $w = f(z) = \phi(x, y)+i\,\psi(x, y)$ is the complex potential
- $\phi(x, y)$ is the velocity potential and $\psi(x, y)$ is the stream function.
- $\phi(x, y) = c_1$ and $\psi(x, y) = c_2$, c_1 and c_2 being constants are respectively called as the equipotential lines and stream lines. These two family of curves intersect each other orthogonally.

Bilinear transformation (BLT)

➢ $w = \dfrac{az+b}{cz+d}$ *where* $ad - bc \neq 0$ *is called a bilinear transformation.*

Invariant (Fixed) points of the BLT are got by solving $w = z$

Cross ratio of a set of four points (P_1 , P_2 , P_3 , P_4) is given by

$$\frac{(P_1 - P_2)(P_3 - P_4)}{(P_2 - P_3)(P_4 - P_1)}$$

Conformal Transformations

➢ *Discussion of standard transformations*

Transformation	*z – plane*	*Image in the w – plane*
$w = e^z$	$x = c_1$	Circle with centre origin
	$y = c_2$	St-line passing through the origin
	(st – lines)	(orthogonal trajectories in the *w-plane)*
$w = z^2$	$x = c_1, -c_1$	Parabola symmetrical about the real axis with vertex $(c_1^2 , 0)$ and focus at the origin.
	$y = c_2, -c_2$ (st- lines)	Parabola symmetrical about the real axis with vertex $(-c_2^2 , 0)$ and focus at the origin.
	$\lvert z \rvert = r$ (circle)	Circle with centre origin and radius r^2.
	$\lvert z - a \rvert = r$	Limacon of the form $r = a + b \cos\theta$.
	and if $r = a$	Cardioide of the form $r = a(1 + \cos\theta)$
$w = z + (k^2/z)$	$\lvert z \rvert = r$ (circle)	Ellipse with foci $(\pm 2k , 0)$.
	amp $z = m$ (st-line)	Hyperbola with foci $(\pm 2k , 0)$. (Confocal conics in the *w*-plane)

Complex Integration

➢ *Complex line integral*

$$\int_C f(z)\, dz = \int_C (u + iv)(dx + i\,dy)$$

$$= \int_C u\,dx - v\,dy + i \int_C v\,dx + u\,dy$$

➤ *Cauchy's theorem*

$$\int_C f(z)\, dz = 0,$$ where $f(z)$ is analytic inside & on a simple closed curve C.

➤ *Cauchy's integral formula*

$$\int_C \frac{f(z)}{z-a}\, dz = 2\pi i f(a)$$

$$\int_C \frac{f(z)}{(z-a)^{n+1}}\, dz = \frac{2\pi i}{n!} f^{(n)}(a)$$

PART - B

Unit - V Special Functions

➤ *Laplace equation in cylindrical system leading to Bessel's differential equation*

$$\frac{\partial^2 f}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial f}{\partial \rho} + \frac{1}{\rho^2}\frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2} = 0$$

➤ *Laplace equation in spherical system leading to Legendre differential equation*

$$\frac{\partial^2 f}{\partial r^2} + \frac{2}{r}\frac{\partial f}{\partial r} + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\,\frac{\partial f}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 f}{\partial \phi^2} = 0$$

Bessel functions

➤ *Bessel differential equation in the standard form*

$$x^2 y'' + x y' + (x^2 - n^2) y = 0$$

Theequation $x^2 y'' + x y' + (\lambda^2 x^2 - n^2) y = 0$ can be reduced to the standard form of Bessel's equation by the substitution $t = \lambda x$

➤ *Bessel function of the first kind*

$$J_n(x) = \sum_{r=0}^{\infty} (-1)^r \left(\frac{x}{2}\right)^{n+2r} \frac{1}{\Gamma(n+r+1)\cdot r!}$$

Property : $J_{-n}(x) = (-1)^n J_n(x)$ where $n = 1, 2, 3, \cdots$

➤ *Two standard results*

$$J_{1/2}(x) = \sqrt{2/\pi x}\ \sin x \ ; \ J_{-1/2}(x) = \sqrt{2/\pi x}\ \cos x$$

The recurrence relation $J_{n-1}(x) + J_{n+1}(x) = \frac{2n}{x} J_n(x)$ helps to find expressions for $J_{3/2}(x), J_{-3/2}(x), J_{5/2}(x)$ etc and also $J_3(x), J_4(x), J_5(x)$ etc.

The recurrence relation $J_n'(x) = \frac{1}{2}\left[J_{n-1}(x) - J_{n+1}(x)\right]$ helps to find / prove several relations involving $J_n''(x)$, $J_n'''(x)$ etc.

➢ *Orthogonal property of Bessel functions*

$$\int_0^1 x\, J_n(\alpha x)\, J_n(\beta x)\, dx = \begin{cases} 0 \text{ if } \alpha \neq \beta \\ \frac{1}{2}[J_n'(\alpha)]^2 \text{ or } \frac{1}{2}[J_{n+1}(\alpha)]^2 \text{ if } \alpha = \beta \end{cases}$$

where α and β are the roots of $J_n(x) = 0$.

Legendre polynomials

➢ *Legendre diferential equation*

$$(1-x^2)y'' - 2xy' + n(n+1)y = 0$$

➢ *Legendre polynomials* $P_n(x)$ *where* $n = 0, 1, 2, 3, 4$ *and* 5.

$$P_0(x) = 1,\ P_1(x) = x,\ P_2(x) = \frac{1}{2}(3x^2 - 1)$$

$$P_3(x) = \frac{1}{2}(5x^3 - 3x),\ P_4(x) = \frac{1}{8}(35x^4 - 30x^2 + 3)$$

$$P_5(x) = \frac{1}{8}(63x^5 - 70x^3 + 15x)$$

➢ *Rodrigue's formula*

$$P_n(x) = \frac{1}{2^n\, n!} \frac{d^n}{dx^n}(x^2 - 1)^n$$

Unit - VI Probability Theorey - 1

➢ *Addition rule*

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

In particular if A and B are mutually exclusive,

$$P(A \cup B) = P(A) + (B)$$

Equivalently $P(A \text{ or } B) = P(A) + P(B)$

➢ *Conditional probability*

$$P(B/A) = \frac{P(A \cap B)}{P(A)} \; ; \; P(A/B) = \frac{P(A \cap B)}{P(B)}$$

➢ *Multiplication rule*

$$P(A \cap B) = P(A) \cdot P(B/A)$$

In particular if A and B are independent,

$$P(A \cap B) = P(A) \cdot P(B)$$

Equivalently $P(A \text{ and } B) = P(A) \cdot P(B)$

➢ *Baye's theorem (rule)*

If $A_1, A_2, \cdots A_n$ are exhaustive and mutually exclusive events of the sample space S and if $A \subset \bigcup_{i=1}^{n} A_i$ then

$$P(A_i/A) = \frac{P(A_i)\,P(A/A_i)}{\sum_{i=1}^{n} P(A_i)\,P(A/A_i)}$$

Unit - VII Probability Theory - 2

➢ *Probability function* $p(x)$

If $p(x_i) \geq 0$ and $\sum_i p(x_i) = 1$ then $p(x)$ is a probability function. The set of values $[x_i, p(x_i)]$ is called a discrete probability distribution of the discrete random variable X.

$P(X)$ is the probability density function. $(p \cdot d \cdot f)$

$f(x) = P(X \leq x) = \sum_{i=1}^{x} p(x_i)$ is called the cumulative distribution function $(c \cdot d \cdot f)$

➢ *Mean* (μ) *and the S.D* (σ)

$$\mu = \sum_i x_i \cdot p(x_i)$$

$$\sigma^2 = \sum_i (x_i - \mu)^2 \cdot p(x_i) = \sum x_i^2\, p(x_i) - [\sum_i x_i p(x_i)]^2$$

➤ *Bernoulli's theorem*

The probability of x successes out of n trials is given by $P(x) = {}^{n}C_{x}\, p^{x} q^{n-x}$

➤ *Discrete probability distributions*

Distribution	Prob. function $P(x)$	Mean (μ)	S.D (σ)
Bionomial	${}^{n}C_{x}\, p^{x} q^{n-x}$	np	$\sqrt{npq}$
Poisson	$\dfrac{m^{x} e^{-m}}{x!}$	m	$\sqrt{m}$

➤ *Continuous probability function $f(x)$*

If $f(x) \geq 0$ and $\int_{-\infty}^{\infty} f(x)\, dx = 1$ then $f(x)$ is called a continuous probability function or probability density function $(p \cdot d \cdot f)$

We have $P(a \leq x \leq b) = \int_{a}^{b} f(x)\, dx$

Also $F(x) = P(X \leq x) = \int_{-\infty}^{x} f(x)\, dx$

is called the cumulative distribution function $(c \cdot d \cdot f)$ of X.

➤ *Mean (μ) and S.D (σ)*

$$\mu = \int_{-\infty}^{\infty} x f(x)\, dx \quad \text{and} \quad \sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\, dx$$

➤ *Continuous probability distributions*

Distribution	Prob. function $f(x)$	Mean	S.D
Exponential	$\alpha e^{-\alpha x}$ for $x > 0$ and 0 otherwise. where $\alpha \neq 0$	$\dfrac{1}{\alpha}$	$\dfrac{1}{\alpha}$
Normal	$\dfrac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$	μ	σ

Mean and S.D of the normal distribution are respectively equal to the mean and S.D of the given distribution.

➢ *Standard Normal Distribution*

- $z = \frac{x-\mu}{\sigma}$ is called the standard normal variate.
- $F(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}$ is the standard normal probability density function or standard normal curve.
- $\phi(z) = \frac{1}{\sqrt{2\pi}} \int_0^z e^{-z^2/2}\, dz$ represents the area under the standard normal curve from 0 to z.

➢ *Some important results*

(i) $P(-\infty \le z \le \infty) = \int_{-\infty}^{\infty} \phi(z)\, dz = 1$

(ii) $P(-\infty \le z \le 0) = \int_{-\infty}^{0} \phi(z)\, dz = 0.5$

(iii) $P(0 \le z \le \infty) = \int_{0}^{\infty} \phi(z)\, dz = 0.5$

(iv) $P(z < z_1) = 0.5 + \phi(z_1)$

(v) $P(z > z_2) = 0.5 - \phi(z_2)$

Unit - VIII Sampling Theory

Sampling Distributions

➢ *Sampling distribution of the means*

	Size	Mean	Variance
Population	N	μ	σ^2
Sample	n	$\mu_{\bar{x}}$	$\sigma_{\bar{x}}^2$

(i) Random sampling with replacement

$\mu_{\bar{x}} = \mu$ and $\sigma_{\bar{x}}^2 = \sigma^2/n$

(ii) Random sampling without replacement

$$\mu_{\bar{x}} = \mu \text{ and } \sigma_{\bar{x}}^2 = \left[\frac{N-n}{N-1}\right]\frac{\sigma^2}{n}$$

➢ *Tests of significance and Confidence intervals*

Acceptance or rejection of the hypothesis

Let μ and σ respectively be the population mean and S.D. Let $\bar{x}$ be the sample mean of a random sample of size n.Then

standard normal variate $z = \dfrac{\bar{x}-\mu}{(\sigma/\sqrt{n})}$

95% confidence interval for μ : $\bar{x} \pm 1.96(\sigma/\sqrt{n})$

99% confidence interval for μ : $\bar{x} \pm 2.58(\sigma/\sqrt{n})$

1.96 and 2.58 are respectively the 95% and 99% confidence coefficients.

➢ *Test of significance of proportions*

Let x be the observed number of successes in a sample size of n and $\mu = np$ be the expected number of successes. The associated $s \cdot n \cdot v$ is given by

$$z = \frac{x-\mu}{\sigma} = \frac{x-np}{\sqrt{npq}}$$

where p is the probability of success and q is the probability of failure.

If $|z| > 2.58$ we conclude that the difference is highly significant and reject the hypothesis at 1% level of significance.

$p \pm 2.58\sqrt{pq/n}$ are the probable limits at 99% confidence level.

Also $p \pm 3\sqrt{pq/n}$ are the probable limits at 99.73% confidence level.

➢ *Test of significance for difference of means*

Let μ_1, μ_2 be the mean of two populations.

Let $(\bar{x}_1, \sigma_1)$, $(\bar{x}_2, \sigma_2)$ be the mean and S.D of two large samples of size n_1 and n_2 respectively. To test the null hypothesis $H_0 : \mu_1 = \mu_2$ the statistic is given by

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\sigma_1^2/n_1 + \sigma_2^2/n_2}}$$

Also the confidence limits for the difference of means of the population are

$$(\bar{x}_1 - \bar{x}_2) \pm z_c \sqrt{\sigma_1^2/n_1 + \sigma_2^2/n_2}$$

where z_c is the confidence coefficient.

➤ *Test of significance for difference of properites (attributes) for two samples*

Let p_1 and p_2 be the sample proportions in respect of an attribute corresponding to 2 large samples of size n_1 and n_2 drawn from two populations. To test the null hypothesis H_0 that there is no difference between the population with regard to the attribute, the statistic is given by

$$z = \frac{p_1 - p_2}{\sqrt{pq\,(1/n_1 + 1/n_2)}}$$

where $p = \dfrac{n_1 p_1 + n_2 p_2}{n_1 + n_2}$ and $q = 1 - p$

➤ *Test of significance for small samples.*

- *Student's t test of a sample mean.*

Statistic $t = \dfrac{\bar{x} - \mu}{s}\sqrt{n}$ where $\bar{x} = \dfrac{1}{n}\sum_{i=1}^{n} x_i$ and $s^2 = \dfrac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$

To test the hypothesis, whether the sample mean $\bar{x}$ differs significantly from the population mean μ, we compute student's t

If $|t| > t_{.05}$ where $t_{.05}$ is the table value of student's t, the difference between $\bar{x}$ and μ is significant at 5% level of significance and the hypothesis is rejected.

Also 95% confidence limits for μ are $\bar{x} \pm \left(\dfrac{s}{\sqrt{n}}\right) t_{.05}$

➤ *Chi - square distribution*

$$\chi^2 = \frac{\sum_{i=1}^{n}(O_i - E_i)^2}{\sum_{i=1}^{n} E_i}$$

where O_i and E_i are respectively the observed and estimated frequencies. As a test of goodness of fit the value of chi-square is used to study the correspondence between these two.

ALPHABETICAL INDEX

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Energy Engineering	P.B.Nagaraj
Foundry Technology	Dr.M.K.Muralidhara
Heat & Mass Transfer	A.Satyanarayana Swamy
	B.N.Veeranna
Hydraulics & Pneumatics	H.D.Ramachandra (HDR)
HTML with Java Script	Satish Hunnur
Industrial Management	K.R.Phaneesh
Introduction to Computer Concepts	Sangamesh.Neela
	Rajashree M.B.
Kinematics of Machines	A.S.Ravindra
Logic Design	Shanmukha Swamy
	R.P.Rajput
Manufacturing Process I	H.D.Ramachandra (HDR)
Manufacturing Process II	H.D.Ramachandra (HDR)
Manufacturing Process III	H.D.Ramachandra (HDR)
Mechatronics	H.D.Ramachandra (HDR)
Material Science & Metallurgy	K.R.Phaneesh
Management & Enterpreneurship	K.R.Phaneesh
Metrology & Measurements	Naidu
Mechanical Measurements & Metrology	P.N.Subramani
OOP with C++	Satish Hunnur
Organisational Management	G.S.Mahajan Shettar
Operations Management	K.R.Phaneesh
Operations Research	K.R.Phaneesh
Operating System	Satish Hunnur
Power Electronics	G.K.Ranganatha Rao
Programming with C	Satish Hunnur
Programming with Jawa	Satish Hunnur
Quality Assurance	Chandrashekar Hiregouder
	B.Raghavendra Reddy
Statistical Quality Control	H.D.Ramachandra (HDR)
Software Engineering	Sangmesh.Neela
Theory of Machines	Ravindra
Thermal Engineering I	G.S.Mahajan Shettar
Thermodynamics Data Hand Book	B.T.Nijaguna & B.S.Samaga
Total Quality Management	H.D.Ramachandra (HDR)
Web Programming	Satish Hunnur